On The Narrow-Band Microwave Filter Design Using a Dielectric Rod

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Abstract —**A method is presented to design bandpass or bandstop filters in the microwave region. The procedure is based on the analysis of the discontinuity problem of a circular cylindrical dielectric rod centered in a rectangular waveguide. For some special relations between the frequency, the dielectric constant, and the radius of the rod, the reflection or the transmission coefficient becomes equal to zero. This relation gives the narrow-band filter. Experimental results for filter design with the help of plastic and porcelain rods are given.**

I. Introduction

IN DESIGNING bandpass or bandstop filters in the microwave region, one usually utilizes a number of sections with obstacles. The most attractive method is to use a single cylindrical post placed in the center of a rectangular waveguide parallel to the electric field of the dominant mode.

There are several interesting papers dealing with the theory of the single-post problem. Marcuvitz's *Waveguide Handbook* [1] provides data for posts which have small radii compared to the waveguide cross section. Lewin [2] has assumed that the radius r of the post is small, allowing higher order terms to be neglected. His assumption helps to find the reflection coefficient for a metallic post. Many other approximations can be found in [3]–[6]. An exact theory has been given by Nielsen [7] in 1969. His theory was applied to a cylinder of arbitrary complex permitivity surrounded by a glass tube. Recently, two excellent papers [8], [9] for the scattering of perfectly conducting [8] and dielectric posts [9] have appeared in the literature. Our geometry is similar to that given in [9], where we have a dielectric post placed centrally in a rectangular waveguide. In our procedure, a theory similar to that given by Nielsen [7] is utilized. The difference between Nielsen's theory and ours is in the geometry of the interaction region. Nielsen used a rectangular interaction region. Unfortunately, after a numerical research, we came to agree with Lewin [10], who expressed his doubts about the validity of Nielsen's theory.

In our method, we assumed a circular interaction region and found a fast convergence in our results. The field was expanded in terms of waveguide modes except in the interaction region where an expansion of cylindrical waves was used.

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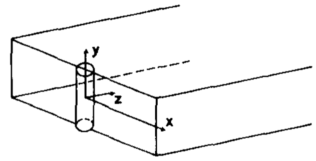

Fig. 1. Cylindrical dielectric post in a rectangular waveguide.

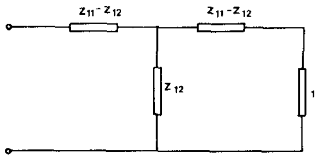

Fig. 2. Equivalent circuit of the rod in an infinite waveguide.

The computer results are in agreement with Marcuvtz's for a small-post radius and have also been extended for a large-post radius where Marcuvitz's expressions are not valid.

The design of a filter is treated by determining the zeros of the reflection and the transmission coefficients for a dielectric post with various permittivities and various radii. After a numerical procedure, we employ some easy to use curves where the resonant frequency of the filter, the dielectric constant, and the radius of the post are related.

II. Formulation

The investigated configuration is shown in Fig. 1. The axis of the dielectric cylinder coincides with the y-axis of the system. It is assumed that the incident wave is the dominant TE_{10} mode traveling in the z-direction. Since we have no variations of the contour of the cylinder in the y-direction, the total electric field will be in the same direction. Our objective is to represent the scattered field in the waveguide. From the field, we can find the reflection and the transmission coefficients which can give the T-equivalent circuit. If R is the reflection and T the transmission coefficient, then by a well-known procedure [8] the expression of the impedance matrix is found.

Using the notation of Fig. 2 [8], we have that

$$\left.\begin{aligned} Z_{11}-Z_{12} &= \frac{(1+R)(1-R)+T^2-2T}{(1-R)^2-T^2} \\ Z_{12} &= \frac{2T}{(1-R)^2-T^2} \end{aligned}\right\} . \quad (1)$$

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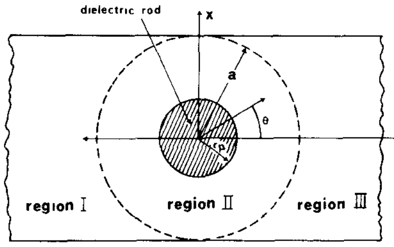

Fig. 3. The regions of the waveguide at the $y = 0$ plane.

The values Z_{11} and Z_{12}, which are of interest in the filter design, depend on the values of the coefficients R and T.

$$\left.\begin{aligned}
&-\sum_{n=0}^{\infty} D_n Z_n\left(\kappa_0 \frac{a}{2}\right)\cos(n\theta_a)+\sum_{m=1}^{\infty} A_m \cos(\gamma_m x_a) e^{-j\kappa_m z_a} = -\cos(\gamma_1 x_a) e^{j\kappa_1 z_a}\\
&-j\sum_{n=0}^{\infty} \kappa_0 D_n Z_n'\left(\kappa_0 \frac{1}{2}\right)\cos(n\theta_a)+\sum_{m=1}^{\infty} A_m\left[\kappa_m \cos(\gamma_m x_a)\cos\theta_a - j\gamma_m \sin(\gamma_m x_a)\sin\theta_a\right] e^{-j\kappa_m z_a}\\
&\qquad\qquad = \left[\kappa_1 \cos(\gamma_1 x_a)\cos\theta_a + j\gamma_1 \sin(\gamma_1 x_a)\sin\theta_a\right] e^{j\kappa_1 z_a}\\
&-\sum_{n=0}^{\infty} D_n Z_n\left(\kappa_0 \frac{a}{2}\right)\cos(n\theta_b)+\sum_{m=1}^{\infty} B_m \cos(\gamma_m x_b) e^{j\kappa_m z_b} = 0\\
&j\sum_{n=0}^{\infty} \kappa_0 D_n Z_n'\left(\kappa_0 \frac{a}{2}\right)\cos(n\theta_b)+\sum_{m=1}^{\infty} B_m\left[\kappa_m \cos(\gamma_m x_b)\cos\theta_b + j\gamma_m \sin(\gamma_m x_b)\sin\theta_b\right] e^{j\kappa_m z_b} = 0
\end{aligned}\right\} \quad (9)$$

We have the following two different cases:

$$R = 0,\ T = e^{jt} \quad (2)$$

$$T = 0,\ R = e^{jr}. \quad (3)$$

Equations (2) and (3) show that the design of a filter can be carried out at the points where the values of the dielectric constant and the radius of the dielectric post make the coefficient R or T equal to zero.

We start analyzing the configuration of Fig. 1 by dividing the waveguide into three separate regions I, II, III (see Fig. 3). The field is expanded in TE_{m0} modes in the regions I and III and in cylindrical modes in the region II (interaction region).

The incoming electric field is

$$E_y^i(x,z) = \cos(\gamma_1 x) e^{j\kappa_1 z} \quad (4)$$

where $\gamma_1 = \pi/a$, $\kappa_1 = (\kappa_0^2 - \gamma_1^2)^{1/2}$, $\kappa_0 = 2\pi/\lambda_0$.

In the outer regions I and III, the field will be expressed as

region I:

$$E_y^I(x,z) = \sum_{m=1}^{\infty} A_m \cos(\gamma_m x) e^{-j\kappa_m z} \quad (5)$$

region III:

$$E_y^{III}(x,z) = \sum_{m=1}^{\infty} B_m \cos(\gamma_m x) e^{j\kappa_m z} \quad (6)$$

where $m = 1,3,5,\ldots$, $\gamma_m = m\pi/\alpha$, $\kappa_m = (\kappa_0^2 - \gamma_m^2)^{1/2}$, and A_m, B_m are arbitrary complex coefficients.

In region II, the field will be expressed separately outside and inside the cylinder. Outside the cylinder, we have

$$E_y^{II}(r,\theta) = \sum_{n=0}^{\infty} \left[C_n J_n(\kappa_0 r) + D_n Y_n(\kappa_0 r)\right]\cos(n\theta)$$

and inside

$$E_y^P(r,\theta) = \sum_{n=0}^{\infty} G_n J_n(\kappa_p r)\cos(n\theta) \quad (8)$$

where C_n, D_n, and G_n are complex expansion coefficients, J_n is the Bessel and Y_n the Neumann function, and κ_p is the propagation constant in the cylinder equal to $\omega\sqrt{\mu\epsilon}$.

Using the Maxwell equations, we can find from (4)–(8) the corresponding components of the magnetic field.

By applying the boundary conditions at the discontinuity surface $r = r_p$ and numerical matching of the field at the surface of the interaction region $r = a/2$ we can find a set of four complex linear equations. These are

where

$$0 < \theta_a \leqq \pi/2 \quad x_a = \frac{a}{2}\sin\theta_a \quad \text{and} \quad Z_a = \frac{a}{2}\cos\theta_a$$

$$0 < \theta_b \leqq \pi/2 \quad x_b = \frac{a}{2}\sin\theta_b \quad \text{and} \quad Z_b = \frac{a}{2}\cos\theta_b$$

$$Z_n(\kappa r) = \frac{J_n(\kappa r)}{g_n(\kappa r)} + Y_n(\kappa r)$$

$$Z_n'(\kappa r) = \frac{J_n'(\kappa r)}{g_n(\kappa r)} + Y_n'(\kappa r)$$

$$g_n = \frac{\kappa_0 J_n'(\kappa_0 r_p) J_n(\kappa_p r_p) - \kappa_p J_n(\kappa_0 r_p) J_n'(\kappa_p r_p)}{\kappa_p Y_n(\kappa_0 r_p) J_n'(\kappa_p r_p) - \kappa_0 Y_n'(\kappa_0 r_p) J_n(\kappa_p r_p)}. \quad (10)$$

For a finite number of terms in the summations and a proper number of different values of θ_a and θ_b, we can solve the above system (9). Our method provides a set of answers for the mode coefficients. If the number of matching points is increased and a negligible change in the coefficient results, it is assumed that the process is convergent and that enough terms have been selected for a given order of accuracy.

This seems a reasonable working assumption if the field representation is itself not defective. In our waveguide structure, the usual series of modes form a complete set and the solution is unique and original.

From all the A_i and B_i, we need the A_1 and B_1 because they give the reflection coefficient R and the transmission coefficient T correspondingly. The A_i and B_i for $i \neq 1$ correspond to image κ_i, which give far from the $z = 0$ point neglecting field terms.

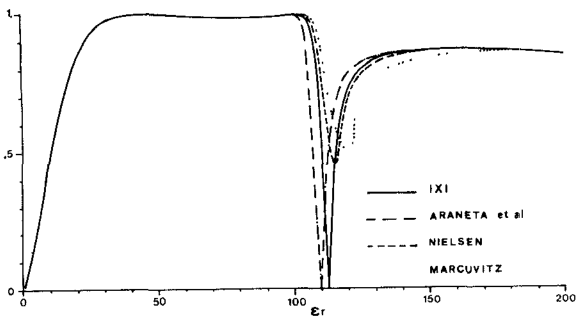

Fig. 4. Magnitude of the reflection coefficient as a function of dielectric constant ($r/a = 0.05$).

It must be pointed out that with the geometrical form of our interaction region we do not need matching conditions on the waveguide walls. This is an advantage of our method compared to Nielsen's, where expression (5) must be valid on the walls.

III. Results and Filter Design

To illustrate our accuracy, let us review the example given by Araneta *et al.* [9]. They compared the results of their technique with those derived by Nielsen and Marcuvitz. Fig. 4 shows the reflection coefficient $|R|$ as a function of the dielectric constant. The relevant parameters are $r/a = 0.05$ and $\lambda_0/\lambda_{c_{1,0}} = 0.7$. λ_0 is the free-space wavelength and $\lambda_{c_{1,0}}$ is the guide-wavelength. Our numerical procedure shows the resonant condition at a dielectric constant of $\epsilon_r = 112.5$. This value was found for different numbers of expansion modes in order to ensure convergence.

The difference between our results and Araneta's is minor for the special case of small-post radius. We believe that our results are more accurate for two reasons.

i) First, because our expressions (7) and (8) with the first two or the first four terms are similar to 1×1 or 2×2 approximations of [9].

ii) Second, because for posts with bigger radius ($r/a > 0.20$) the 2×2 approximation gives pure results. (In some cases, we found $|R| > 1$.)

Our results with a 6×6 approximation is a logical conclusion to that given in [9], where for a 2×2 approximation the dip in $|R|$ compared to that given for 1×1 approximation goes to increases ϵ_r.

Fig. 5(a) and 5(b) show the results of the reflection and the transmission coefficients for a classical case of a wavelength ratio $\lambda_0/\lambda_{c_{1,0}}$ equal to 0.799 and $r/a = 0.1$. The results show a reverse variation between the $|R|$ and $|T|$ versus the dielectric constant. To design a filter, we need a post with a dielectric constant equal to 34.4. In this case, the reflection coefficient is zero and the transmission has a value equal to

$$T = e^{\pm j\pi}. \tag{11}$$

This gives that $Z_{11} - Z_{12} = 0$, $Z_{12} = \infty$ and the filter will be a bandpass filter.

An estimation of the accuracy shows the absorption coefficient $A = \sqrt{1 - |R|^2 - |T|^2}$, which must be zero. In Table I, we show the values of R, T, and A^2 for a post with $r/a = 0.25$ and $\lambda_0/\lambda_{c_{1,0}} = 0.6366$.

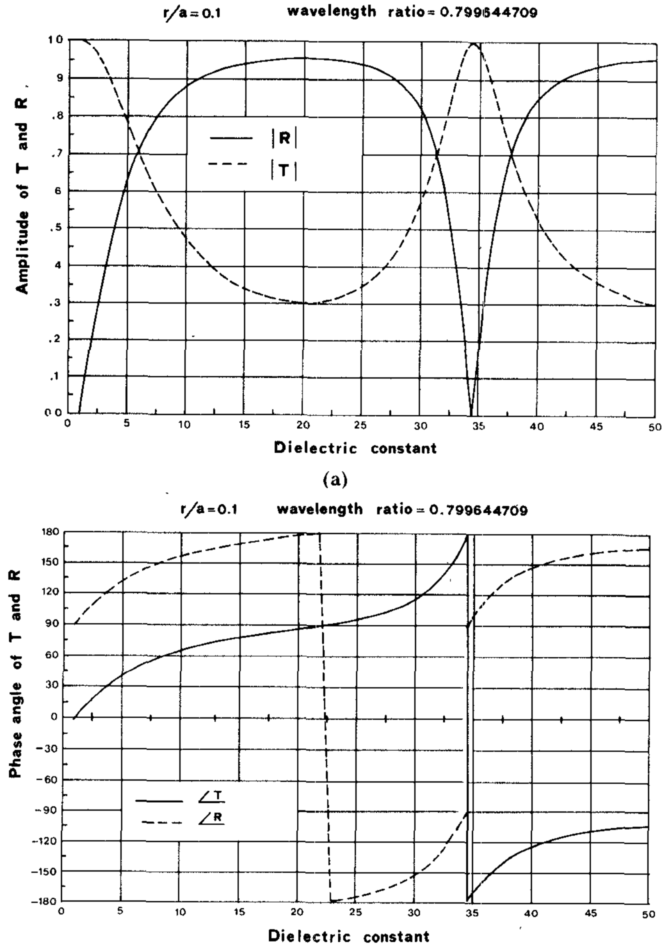

Fig. 5. (a) Magnitude of the reflection and transmission coefficient as a function of dielectric constant ($r/a = 0.1$). (b) Phase of the reflection and transmission coefficient as a function of dielectric constant ($r/a = 0.1$).

As we can see, there is no convergence in the 2×2 approximation while we get accurate results for the 6×6 and even more accurate results for the 9×9 approximation.

Fig. 6 shows the amplitudes of the reflection and the transmission coefficients for the same post. It is shown that there are many min and max, and we could design filters for dielectric constants given in Table II.

From the results given before, we conclude that there is always a combination between r, a, $\lambda_{c_{1,0}}$, and λ_0 for which we could design a narrow-band filter. With our method, we can find all the forms of the filter for any kind of dielectric post.

A numerical procedure gave a relation of the form

$$r_p/a = f(\epsilon_r, \lambda_0/\lambda_{c_{1,0}}). \tag{12}$$

For a bandpass filter, the simpler relation is of the form

$$r_p/a = A(\lambda_0/\lambda_{c_{1,0}})\epsilon_r^{-B(\lambda_0/\lambda_{c1,0})} \tag{13}$$

and for a bandstop filter of the form

$$r_p/a = C(\lambda_0/\lambda_{c_{1,0}})\epsilon_r^{-D(\lambda_0/\lambda_{c1,0})}. \tag{14}$$

The functions $A(\lambda_0/\lambda_{c_{1,0}})$, $B(\lambda_0/\lambda_{c_{1,0}})$, $C(\lambda_0/\lambda_{c_{1,0}})$, and $D(\lambda_0/\lambda_{c_{1,0}})$ are given in the Figs. 7(a) and (b) and 8(a) and (b). Since we have more than one resonance, there are

TABLE I
REFLECTION, TRANSMISSION, AND ABSORPTION COEFFICIENTS FOR DIFFERENT VALUES OF THE DIELECTRIC CONSTANT FOR A POST WITH $r/a = 0.25$ AND $\lambda_0/\lambda_{c_{1,0}} = 0.6366$

ε_r	2 x 2 approximation			6 x 6 approximation			9 x 9 approximation		
	|R|	|T|	A^2	|R|	|T|	A^2	|R|	|T|	A^2
9.75	1.733119	0.904559	-2.821928	0.818899	0.594025	-0.023461	0.814690	0.579856	4.7223×10^{-5}
9.80	0.629549	0.964953	-0.327466	0.629455	0.764319	-0.027156	0.637145	0.770739	7.6429×10^{-6}
9.85	0.270158	0.448906	0.725498	0.453720	0.879967	0.019796	0.460899	0.887464	-2.0240×10^{-5}
9.90	0.267652	0.967712	0.008104	0.301495	0.948478	0.009490	0.305940	0.952054	-6.1030×10^{-6}
9.95	0.203526	0.895640	0.156406	0.175139	0.980196	0.008542	0.175591	0.984462	2.3713×10^{-6}
10.00	0.278382	0.952654	0.014954	0.073223	0.992840	0.008907	0.066800	0.997766	7.6920×10^{-7}

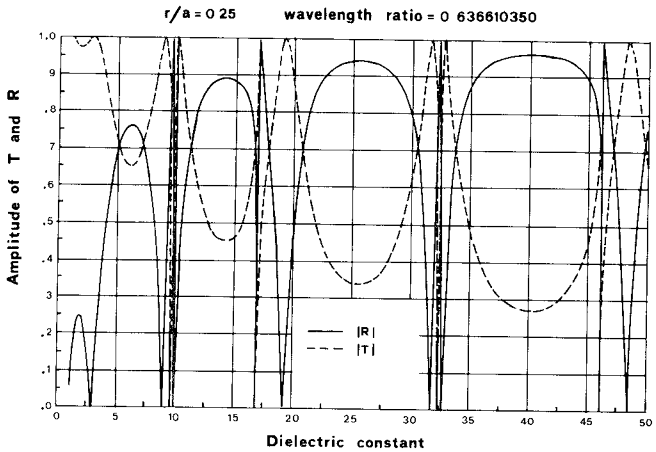

Fig. 6. Magnitude of the reflection and transmission coefficient as a function of dielectric constant ($r/a = 0.25$).

TABLE II
FILTERS FOR DIFFERENT VALUES OF THE DIELECTRIC CONSTANT FOR A POST WITH $r/a = 0.25$ AND $\lambda/\lambda_{c_{1,0}} = 0.6366$

ε_r	|R|	|T|	Kind of filter
2.95	0	1	B.P.
9.03	0	1	B.P.
9.65	1	0	B.S.
10.02	0	1	B.P.
16.87	0	1	B.P.
17.03	1	0	B.S.
19.20	0	1	B.P.
31.72	0	1	B.P.
32.34	1	0	B.S.
32.72	0	1	B.P.
46.11	0	1	B.P.
46.25	1	0	B.S.
48.43	0	1	B.P.

more than one curve that is a function of A, B, C, and D. The first three curves are given in Fig. 7(a) and (b). It is interesting to note that, in all cases, the functions B and D are near the value 0.5.

From the curves in Figs. 7 and 8, we can design a filter for a given resonance frequency. If we have a dielectric

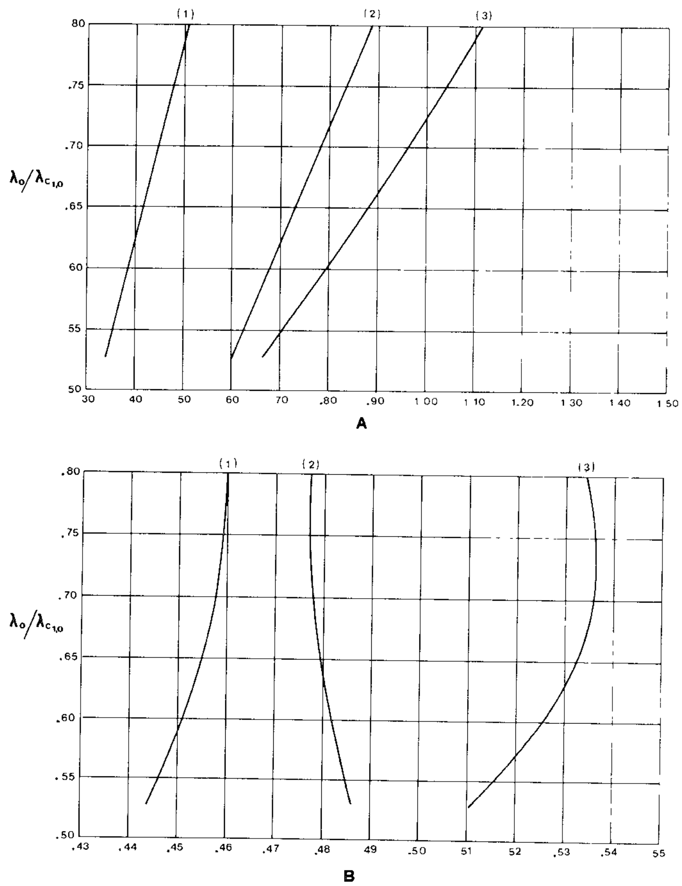

Fig. 7. (a) Coefficient A as a function of $\lambda_0/\lambda_{c_{1,0}}$. (b) Coefficient B as a function of $\lambda_0/\lambda_{c_{1,0}}$

material, we can define the radius of the post, while , while for a given radius, we can define the dielectric constant.

As a simple example, we give the design of a bandstop filter for a WR 90 waveguide in the frequency of 10.301 GHz. From Fig. 8(a) and (b), we have that $c = 0.821$ and $D = 0.5245$. To have a ratio $r_p/a = 0.25$, we need a dielectric constant equal to 9.64672. In this case, the insertion loss of the filter as a function of frequency is given in Fig. 9.

For a resonant frequency of 10.380 GHz, the dielectric constant will become equal to 9.472. So, we see that a small difference in the dielectric constant can markedly change the resonant frequency.

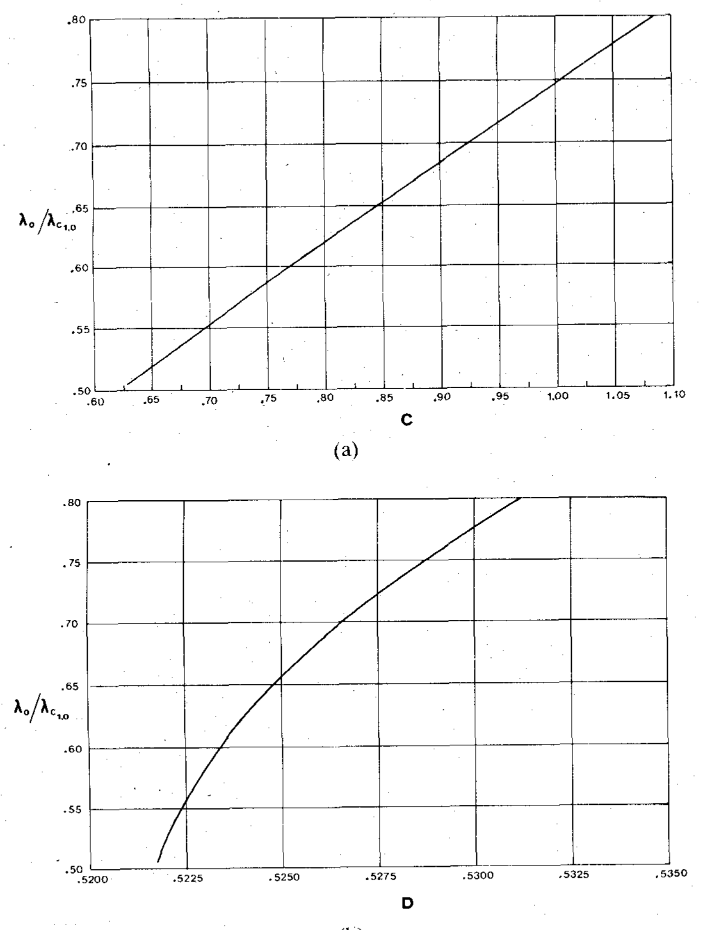

Fig. 8. (a) Coefficient C as a function of $\lambda_0/\lambda_{c_{1,0}}$. (b) Coefficient D as a function of $\lambda_0/\lambda_{c_{1,0}}$.

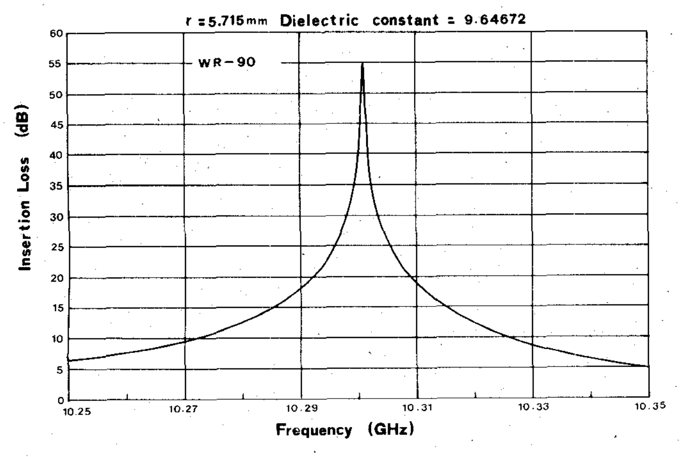

Fig. 9. Theoretical response of a bandstop filter.

A question which remain is the problem of the dielectric loss. Have we the same resonant frequency for a dielectric post with losses and without losses?. A numerical investigation gives the same results as expected. We have the same resonant frequency but the insertion-loss curve will be smoother. Fig. 10 shows the insertion loss as a function of frequency for a post with $r_p/a = 0.25$ and $\epsilon_r = 9.472$ for three different $\tan\delta$.

Since it is not possible or even practical to make materials with relative dielectric constants to the accuracy required, our approach must start from a given material

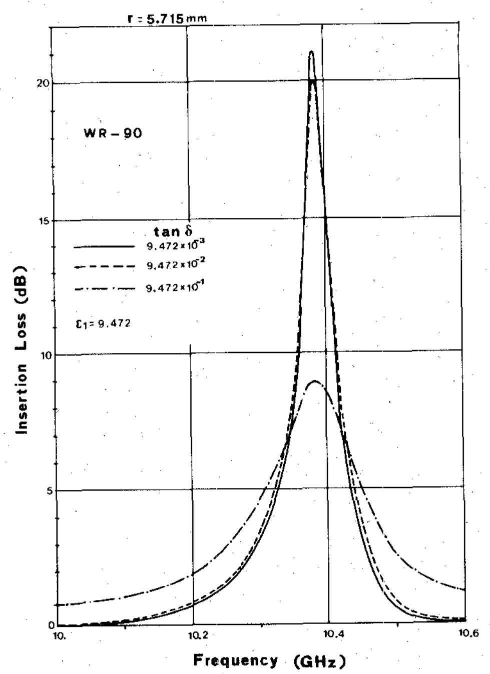

Fig. 10. Theoretical response of a bandstop filter with a dielectric rod for various $\tan\delta$.

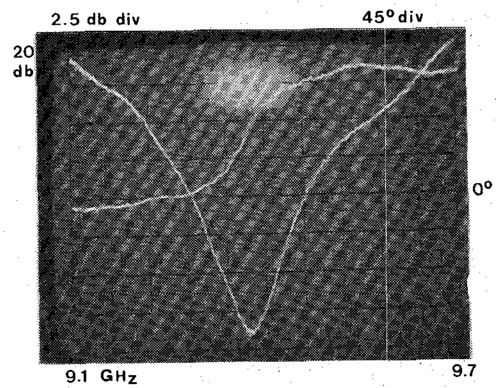

Fig. 11. Measured response of a bandstop filter with a plastic (ertalon) rod.

which will be used to build the dielectric post. The radius of the post can be made to the accuracy required, after a definition from (13) or (14).

The above examples show that it is possible to design narrow-band filters with the help of dielectric posts.

To show the accuracy of the method, an experimental filter was designed, fabricated, and evaluated. The post was designed from the plastic material ertalon with a dielectric constant $\epsilon_1 = 3.12$ and resonant frequency $f = 9.37$ GHz for the WR 90 waveguide. Fig. 7(a) and (b) given that $A = 0.447$ and $B = 0.4575$, and from those we get a $r_p/a = 0.26575$. An experimental verification with the help of the HP-8410B network analyzer gives the return loss of the filter as a function of frequency, which shows the resonance at the same frequency (see Fig. 11).

A porcelain rod with a dielectric constant $\epsilon_1 = 5.446$ and $\tan\delta = 2.33 \cdot 10^{-2}$ was also evaluated. To have a stopband at the frequency 12.4651 GHz, we found that $C = 0.6613$ and $D = 0.5221$. Equation (14) gave a ratio r_p/a equal to 0.27296. The theoretical and measured performance of the

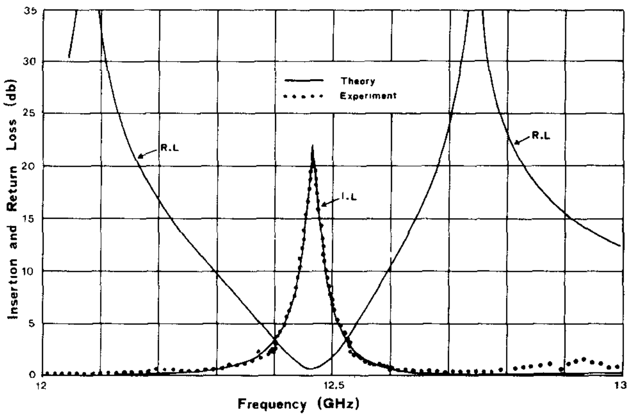

Fig. 12. Theoretical and measured response of a bandstop filter.

filter is shown in Fig. 12. As we can see, there is excellent agreement between the theoretical and the experimental results.

IV. A Theoretical Expression of the Resonant Condition

The procedure presented gives numerically the values of the reflection and transmission coefficients. Close scrutiny of (9) gives some further ideas on the theoretical definition of the resonant conditions.

If we change the angle θ_a to $\theta = \pi - \theta_a$ and θ_b to θ, we get only one variable θ where $0 \leq \theta \leq \pi/2$

$$\left.\begin{aligned}
&-\sum_{n=0}^{\infty} D_n Z_n\left(\kappa_0 \frac{a}{2}\right)(-1)^n \cos(n\theta) + \sum_{m=1}^{\infty} A_m \cos(\gamma_m x) e^{-j\kappa_m z} = -\cos(\gamma_1 x) e^{-j\kappa_1 z}\\
&j\sum_{n=0}^{\infty} \kappa_0 D_n Z_n'\left(\kappa_0 \frac{a}{2}\right)(-1)^n \cos(n\theta) + \sum_{n=1}^{\infty} A_m\left[\kappa_m \cos(\gamma_m x)\cos\theta + j\gamma_m \sin(\gamma_m x)\sin\theta\right] e^{j\kappa_m z}\\
&\qquad = \left[\kappa_1 \cos(\gamma_1 x)\cos\theta - j\gamma_1 \sin(\gamma_1 x)\sin\theta\right] e^{-j\kappa_1 z}\\
&-\sum_{n=0}^{\infty} D_n Z_n\left(\kappa_0 \frac{a}{2}\right)\cos(n\theta) + \sum_{m=1}^{\infty} B_m \cos(\gamma_m x) e^{j\kappa_m z} = 0\\
&j\sum_{n=0}^{\infty} \kappa_0 D_n Z_n'\left(\kappa_0 \frac{a}{2}\right)\cos(n\theta) + \sum_{m=1}^{\infty} B_m\left[\kappa_m \cos(\gamma_m x)\cos\theta + j\gamma_m \sin(\gamma_m x)\sin\theta\right] e^{j\kappa_m z} = 0
\end{aligned}\right\} \quad (15)$$

where $x = a/2 \sin\theta$ and $z = a/2 \cos\theta$.

After some algebra, we split the even and odd terms of D_n in the following system:

$$\left.\begin{aligned}
&-2\sum_{n=0}^{\infty} D_{2n} Z_{2n}\left(\kappa_0 \frac{a}{2}\right)\cos(2n\theta) + \sum_{n=1}^{\infty}(A_m + B_m)\cos(\gamma_m x) e^{j\kappa_m z} = -\cos(\gamma_1 x) e^{-j\kappa_1 z}\\
&2\sum_{n=1}^{\infty} D_{2n-1} Z_{2n-1}\left(\kappa_0 \frac{a}{2}\right)\cos(2n-1)\theta + \sum_{n=1}^{\infty}(A_m - B_m)\cos(\gamma_m x) e^{j\kappa_m z} = -\cos(\gamma_1 x) e^{-j\kappa_1 z}\\
&2j\sum_{n=0}^{\infty} \kappa_0 D_{2n} Z_{2n}'\left(\kappa_0 \frac{a}{2}\right)\cos(2n\theta) + \sum_{n=1}^{\infty}(A_m + B_m)\left[\kappa_m \cos(\gamma_m x)\cos\theta + j\gamma_m \sin(\gamma_m x)\sin\theta\right] e^{j\kappa_m z}\\
&\qquad = \left[\kappa_1 \cos(\gamma_1 x)\cos\theta - j\gamma_1 \sin(\gamma_1 x)\sin\theta\right] e^{-j\kappa_1 z}\\
&-2j\sum_{n=1}^{\infty} \kappa_0 D_{2n-1} Z_{2n-1}'\left(\kappa_0 \frac{a}{2}\right)\cos(2n-1)\theta + \sum_{n=1}^{\infty}(A_m - B_m)\left[\kappa_m \cos(\gamma_m x)\cos\theta + j\gamma_m \sin(\gamma_m x)\sin\theta\right] e^{j\kappa_m z}\\
&\qquad = \left[\kappa_1 \cos(\gamma_1 x)\cos\theta - j\gamma_1 \sin(\gamma_1 x)\sin\theta\right] e^{-j\kappa_1 z}
\end{aligned}\right\} \quad (16)$$

Multiplying by $\cos m\theta$, we get

$$D_{2n} = \frac{2}{\pi Z_{2n}\left(\kappa_0 \frac{a}{2}\right)} \cdot \left\{R_{-1,2n} + \sum_{n=1}^{\infty}(A_m + B_m) R_{m,2n}\right\}$$

$$D_{2n} = -\frac{2j}{\pi Z_{2n}'\left(\kappa_0 \frac{a}{2}\right)} \cdot \left\{L_{-1,2n} - \sum_{n=1}^{\infty}(A_m + B_m) L_{m,2n}\right\}$$

$$D_{2n-1} = \frac{2}{\pi Z_{2n-1}\left(\kappa_0 \frac{a}{2}\right)} \cdot \left\{-R_{-1,2n-1} - \sum_{n=1}^{\infty}(A_m - B_m) R_{m,2n-1}\right\}$$

$$D_{2n-1} = -\frac{2}{\pi Z_{2n-1}'\left(\kappa_0 \frac{a}{2}\right)} \cdot \left\{L_{-1,2n-1} + \sum_{n=1}^{\infty}(A_m - B_m) L_{n,2n-1}\right\} \quad (17)$$

where

$$R_{\pm n,m} = \int_0^{\pi/2} \cos(\gamma_n x) e^{\pm j\kappa_n z} \cos m\theta \, d\theta$$

$$L_{\pm n,m} = \frac{1}{\kappa_0} \int_0^{\pi/2} [\kappa_n \cos(\gamma_n x) \cos\theta + j\gamma_n \sin(\gamma_n x) \sin\theta] \cdot e^{\pm j\kappa_m z} \cos m\theta \, d\theta. \quad (18)$$

Equating the even and odd D_n, we have that

$$\sum_{m=1}^{\infty} (A_m + B_m)\left[Z'_{2n}\left(\kappa_0 \frac{a}{2}\right) R_{m,2n} - jZ_{2n}\left(\kappa_0 \frac{a}{2}\right) L_{m,2n}\right] = -\left[Z'_{2n}\left(\kappa_0 \frac{a}{2}\right) R_{-1,2n} + jZ_{2n}\left(\kappa_0 \frac{a}{2}\right) L_{-1,2n}\right] \quad (19)$$

$$\sum_{m=1}^{\infty} (A_m - B_m)\left[Z'_{2n-1}\left(\kappa_0 \frac{a}{2}\right) R_{m,2n-1}\left(\kappa_0 \frac{a}{2}\right) L_{m,2n-1}\right] = -\left[Z'_{2n-1}\left(\kappa_0 \frac{a}{2}\right) R_{-1,2n-1} + jZ_{2n-1}\left(\kappa_0 \frac{a}{2}\right) L_{-1,2n}\right]. \quad (20)$$

For a finite number of terms in (19) and (20) and for different values of n, we can have two systems of linear equations with unknowns $(A_m + B_m)$ and $(A_m - B_m)$. From those, we look at the $(A_1 + B_1)$ and $(A_1 - B_1)$. The condition

$$(A_1 + B_1) = (A_1 - B_1) \quad (21)$$

gives a bandstop filter, while the

$$(A_1 + B_1) = -(A_1 - B_1) \quad (22)$$

gives a bandpass. Both $(A_1 + B_1)$ and $(A_1 - B_1)$ depend on $Z_n(\kappa_0 a/2)$, $Z'_n(\kappa_0 a/2)$, $R_{m,n}$, $L_{m,n}$, and from (21) and (22) we can find the resonant condition by changing the ϵ_r, r_p, and κ_0.

We could use the system (19) and (20) to find all the coefficients A_m and B_m. Our preference on the point-matching technique came from the expression of the inner products (18) for which there is not any easy analytic expression.

V. Conclusion

A numerical method has been given to analyze a dielectric post in the middle of a rectangular waveguide. From the method, a technique to design narrow-band filters was presented. The filter design was given, in easy to use graphical form, for bandpassing and bandstoping. An experimental result has shown the validity of the procedure.

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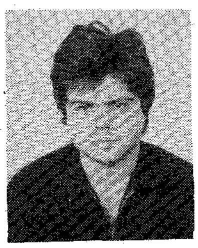

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